Lección de Cálculo Avanzado para Físicos.

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Cálculo de Variaciones

$$F = F(x, y, y')$$

queremos minimizar $I = \int_R F(x, y, y')dx$

entonces hacemos una variación dependiendo del parámetro ϵ esto es

$$I(\epsilon) = \int_R F(x, y + \epsilon\eta, y' + \epsilon\eta')dx$$

note que

$$I(0) = \int_R F(x, y, y')dx$$

expandamos el integrando

$$F(x, y + \epsilon\eta, y' + \epsilon\eta') = F(x, y, y') + \frac{\partial F(x, y, y')}{\partial y}\epsilon\eta + \frac{\partial F(x, y, y')}{\partial y'}\epsilon\eta' + o(\epsilon^2)$$

o bien

$$F(x, y + \epsilon\eta, y' + \epsilon\eta') - F(x, y, y') = \epsilon\eta\frac{\partial F(x, y, y')}{\partial y} + \epsilon\eta'\frac{\partial F(x, y, y')}{\partial y'} + o(\epsilon^2)$$

$$\frac{F(x, y + \epsilon\eta, y' + \epsilon\eta') - F(x, y, y')}{\epsilon} = \eta\frac{\partial F(x, y, y')}{\partial y} + \eta'\frac{\partial F(x, y, y')}{\partial y'} + \frac{o(\epsilon^2)}{\epsilon}$$

$$\int_R \frac{F(x, y + \epsilon\eta, y' + \epsilon\eta') - F(x, y, y')}{\epsilon}dx = \int_R \eta\frac{\partial F(x, y, y')}{\partial y}dx + \int_R \eta'\frac{\partial F(x, y, y')}{\partial y'}dx + \int_R \frac{o(\epsilon^2)}{\epsilon}dx$$

$$\lim_{\epsilon\to 0}\int_R \frac{F(x, y + \epsilon\eta, y' + \epsilon\eta') - F(x, y, y')}{\epsilon}dx = \int_R \eta\frac{\partial F(x, y, y')}{\partial y}dx + \int_R \eta'\frac{\partial F(x, y, y')}{\partial y'}dx + \lim_{\epsilon\to 0}\int_R \frac{o(\epsilon^2)}{\epsilon}dx$$

$$\frac{dI}{d\epsilon} = \int_R \eta\frac{\partial F(x, y, y')}{\partial y}dx + \int_R \frac{\partial F(x, y, y')}{\partial y'}\eta' dx$$

$$\frac{dI}{d\epsilon} = \int_R \eta\frac{\partial F}{\partial y}dx + \int_R \eta'\frac{\partial F}{\partial y'}dx$$

$$\frac{dI}{d\epsilon} = \int_R \eta\frac{\partial F}{\partial y}dx - \int_R \eta\frac{d}{dx}\frac{\partial F}{\partial y'}dx$$

$$\frac{dI}{d\epsilon} = \int_R \eta\left(\frac{\partial F}{\partial y} - \frac{d}{dx}\frac{\partial F}{\partial y'}\right)dx$$

pero queremos

$$\frac{dI}{d\epsilon}(0) = 0$$

entonces

$$\int_R \eta\left(\frac{\partial F}{\partial y} - \frac{d}{dx}\frac{\partial F}{\partial y'}\right)dx = 0$$

para cada elección de η, entonces

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\frac{\partial F}{\partial y'} = 0$$